where x, y, and t are integers, x and y have opposite parity, $x > y$, and a and b are the legs of the triangle. Using this representation of the triangle sides, and letting P be the perimeter of the triangle and A the area, we have $P = a + b + c = 2xyt + (x^2 - y^2)t + (x^2 + y^2)t = 2xt(x + y)$ and $A = \frac{1}{2}ab = \frac{1}{2}(2xyt)(x^2 - y^2)t = xyt^2(x^2 - y^2)$. Now the problem stipulates that the area is λ times the perimeter, which implies that

$$xyt^2(x^2 - y^2) \;=\; \lambda\bigl(2xt(x + y)\bigr)\,.$$

This equation can be solved for λ to yield $\lambda = \frac{1}{2}y(x - y)t$. Such λ will be an integer unless both t and y are odd. Therefore, all solutions are given by

$$(a, b, c) \;=\; \left(2xyt,\; (x^2 - y^2)t,\; \frac{y(x - y)t}{2}\right)\,,$$

where x and y have opposite parity, $x > y$, and at least one of y and t is even.

One incomplete solution was received.

M150. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let two complex numbers z_1 and z_2 satisfy the conditions

$$\begin{aligned} z_1 + z_2 &= -(i + 1)\,, \\ z_1 \cdot z_2 &= -i\,. \end{aligned}$$

Without calculating z_1 and z_2, find $z_1 \cdot \overline{z_2}$.

Solution by the proposer.

Note that $z_1 \cdot \overline{z_2} = \frac{z_1}{z_2} \cdot |z_2|^2$. From $(z_1 + z_2)^2 = 2i = -2z_1 \cdot z_2$, we immediately obtain $z_1^2 + 4z_1z_2 + z_2^2 = 0$, or equivalently,

$$\left(\frac{z_1}{z_2}\right)^2 + 4\left(\frac{z_1}{z_2}\right) + 1 \;=\; 0\,.$$

Thus, $\frac{z_1}{z_2}$ is real and negative. Therefore, $z_1 \cdot \overline{z_2}$ is also real and negative. Combining this with $|z_1 \cdot \overline{z_2}| = |z_1 \cdot z_2| = 1$, we see that $z_1 \cdot \overline{z_2} = -1$.